# Higher-Order Eggshell Method for Computation of Forces Acting on Ferromagnetic Bodies

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## Abstract

An alternative higher-order Eggshell method is proposed for determining forces acting on a ferromagnetic body in magnetic field. The strongly nonlinear problem is solved by the Newton method effectively implemented in our codes Agros2D and Hermes. The computation exhibits substantially higher parameters (accuracy, time of solution) than the algorithms used even in available professional codes. The methodology is illustrated with a typical example.

## 1 Introduction

The computation of forces acting on ferromagnetic bodies in stronger magnetic fields still belongs to relatively difficult tasks. Although there exist two basic approaches to their evaluation (virtual work and Maxwell stress tensor), due to high nonlinearities the computations are relatively slow, convergence is rather poor and accuracy of the results is often insufficient. The paper offers an alternative technique based on a higher-order Eggshell method whose algorithms are implemented in our own software Agros2D and Hermes.

The power of the method is demonstrated on an example of computation of the force acting on the plunger of a DC electromagnet. Its principal arrangement (together with the main dimensions) is shown in Fig. 1.

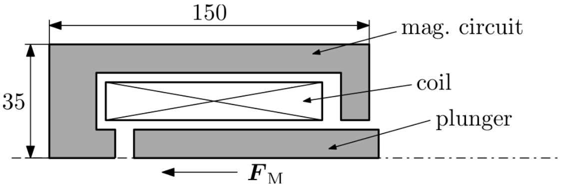

Fig. 1: Principal scheme of DC electromagnet

The coil containing 1200 turns is supplied by direct current $I = 3\,\mathrm{A}$, which is enough for reaching a high saturation of the magnetic circuit. All necessary material parameters and magnetization characteristics are known.

## 2 Continuous mathematical models

The distribution of a stationary magnetic field is described by the equation for the magnetic vector potential $\boldsymbol{A}$

$$\mathrm{curl}\,\frac{1}{\mu(|\boldsymbol{B}|)}\,\mathrm{curl}\,\boldsymbol{A} = \boldsymbol{J}_\mathrm{ext}, \quad \boldsymbol{B} = \mathrm{curl}\,\boldsymbol{A}, \tag{1}$$

where $\mu$ stands for the magnetic permeability, $\boldsymbol{J}_\mathrm{ext}$ is the field current density and $\boldsymbol{B}$ denotes the vector of the magnetic flux density. The field distribution depends on the current position of the plunger. The permeability exhibits a very strong nonlinear dependence on magnetic flux density $|\boldsymbol{B}|$, which causes the convergence problems.

### 2.1 Computation of forces from Virtual Work

The magnetic energy $W_\mathrm{M}$ in the system follows from the formulae

$$w_\mathrm{M} = \int_0^{\boldsymbol{B}} \boldsymbol{H}\,\mathrm{d}\boldsymbol{B}, \quad W_\mathrm{M} = \int_V w_\mathrm{M}\,\mathrm{d}V, \tag{2}$$

where $w_\mathrm{M}$ is the local density of energy and $V$ is its volume. The total force acting then follows from the formula

$$\boldsymbol{F}_\mathrm{M} = -\mathrm{grad}\,W_\mathrm{M}. \tag{3}$$

### 2.2 Computation of forces using Maxwell's Stress Tensor

The Maxwell stress tensor can be written in the form

$$\sigma_\mathrm{M} = -\frac{1}{2\mu}\left(\boldsymbol{B}\cdot\boldsymbol{B}\right)\boldsymbol{I} + \frac{1}{\mu}\boldsymbol{B}\boldsymbol{B}^\mathrm{T}. \tag{4}$$

where $\boldsymbol{I}$ is the unit matrix. The force $\boldsymbol{F}_\mathrm{M}$ acting on the body (plunger) is now given by the integral

$$\boldsymbol{F}_\mathrm{M} = \oint_S \sigma_\mathrm{M}\,\mathrm{d}\boldsymbol{S}, \tag{5}$$

where $\boldsymbol{S}$ denotes the outward unit normal vector to the boundary $S$ of the plunger.

### 2.3 Eggshell method

The Eggshell method [1] is based on introducing a thin shell over the surface of the body (see Fig. 2) and an appropriate function $\gamma$ satisfying the conditions $\gamma = 0$ along the external boundary of the shell and $\gamma = 1$ along its internal boundary. Then the magnetic force acting on the body is

$$\boldsymbol{F}_\mathrm{M} = \int_V \sigma_\mathrm{M}\cdot\mathrm{grad}\,\gamma\,\mathrm{d}V, \tag{6}$$

where $V$ is the volume of the eggshell. Function $\gamma$ can be chosen in more ways. In our implementation, we use the solution of the Laplace equation in the eggshell domain obtained using higher-order finite elements. These bring additional smoothness to the solution and we can choose their orders arbitrarily. Since the eggshell domain is covered with very few elements, the additional computational cost caused by increasing the element order is quite negligible.

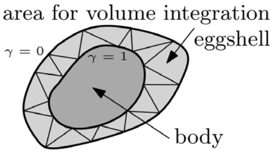

Fig. 2: Principal scheme of Eggshell method

## 3 Numerical Solution

The task was solved by our own code Agros2D [2] based on a fully adaptive higher-order finite element method [3]. The problems with strong nonlinearities were overcome by our effective implementation of the Newton algorithm.

Figure 3 shows a part of the eggshell surrounding the end of the plunger.

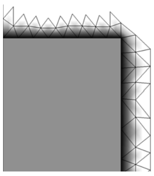

Fig. 3: Part of eggshell around plunger of actuator

Table 1 contains the parameters of computation needed for determining the forces at three positions of the plunger (0.01, 0.02 and 0.03 m from the left end), i.e., in highly saturated states of the device. The number of DOFs was comparable, the prescribed residual norm was put $10^{-4}$ .

Table 1: Comparison of parameters of computation using COMSOL and Agros2D for three positions of the plunger

| code | COMSOL (Dogleg) | | | Agros2D | | |
|---|---|---|---|---|---|---|
| position (m) | 0.01 | 0.02 | 0.03 | 0.01 | 0.02 | 0.03 |
| DOFs | 27938 | 28121 | 28046 | 27208 | 27529 | 28876 |
| time (s) | 73 | 153 | 35 | 10 | 19 | 9 |
| iterations | 132 | 275 | 62 | 21 | 27 | 20 |
| force (N) | 14.41 | 10.36 | 8.76 | 13.23 | 10.09 | 9.66 |

Figure 4 compares several methods of calculation of the static characteristic with measurement on the experimental device. The computations are performed for lower (but again mutually comparable) numbers of DOFs than in Tab. 1. Finally, Fig. 5 shows the static characteristic computed by Agros using the Maxwell tensor and two (first-order and third-order) Eggshell methods. It is clear that even the results obtained by the first-order Eggshell method are much better.

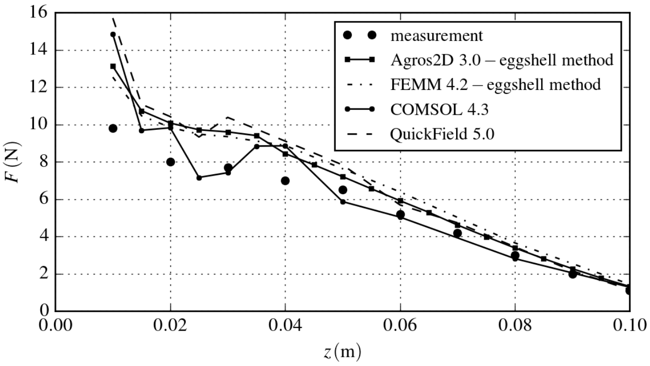

Fig. 4: Comparison of several methods for calculating force acting on armature of electromagnetic actuator

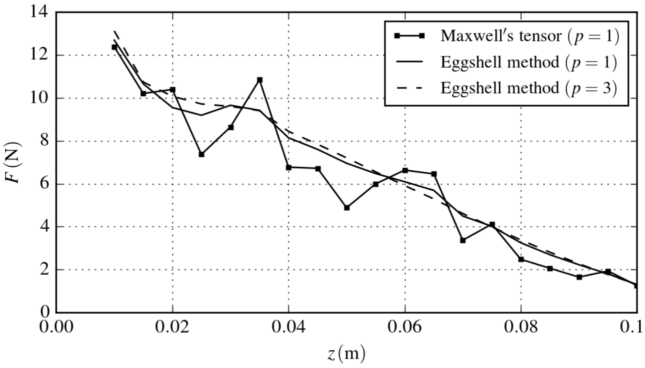

Fig. 5: Comparison of first-order Maxwell stress tensor, first-order eggshell method and third-order eggshell method

Computation of magnetic forces in several other arrangements by the above algorithm confirms its excellent properties.

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## References

[1] F. Henrotte, M. Felden, M. vander Giet and K. Hameyer. “The eggshell approach for the computation of electromagnetic forces in 2D and 3D", COMPEL: The International Journal for Computation and Mathematics in Electrical and Electronic Engineering, Vol. 23, No. 4, pp. 996–1005, 2004.

[2] P. Karban, F. Mach, P. Kůs, D. Pánek, I. Doležel. “Numerical solution of coupled problems using code Agros2D”, Computing, Vol. 95, No. 1, Supplement, pp 381–408, DOI 10.1007/s00607-013-0294-4, 2013.

[3] P. Solin, L. Korous. “Adaptive *hp*-FEM with dynamical meshes for problems with travelling sharp fronts”, Computing, published online: DOI 10.1007/s00607-012-0243-7, 2012.